

# Fine structure and hyperfine structure of some excited states of helium

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**Abstract.** The fine structure and the hyperfine structure for some singly-excited and doubly-excited states of helium atom are calculated using Rayleigh-Ritz variational method with multi-configuration-interaction wave functions. The calculated results of the fine structure for the Rydberg series are in good agreement with other theoretical and experimental data. The hyperfine parameters and the hyperfine coupling constants of <sup>3</sup>He are also obtained for this system.

**PACS.** 32.10.Fn Fine and hyperfine structure – 31.30.Gs Hyperfine interactions and isotope effects, Jahn-Teller effect

## 1 Introduction

Helium is the simplest coulomb three-body system that exhibits strong electron-electron correlation. Both experimental measurements of energies and spectra in helium and the corresponding theoretical calculations are fundamental problems in atomic physics. The large fine structure intervals of the  $1s2p\ ^3P^o$  state of helium is ideal for investigating the fine structure constant  $\alpha$  and for testing QED to this two-electron system. Many experimental and theoretical investigations have been achieved very high precision for the  $1s2p\ ^3P^o$  state [1–17]. Experimentally, the development of laser spectroscopy and microwave measurement made advantageous to measure fine structure to very high accuracy. Shiner *et al.* [1] measured the Lamb shift and fine structure of the  $2s\ ^3S-2p\ ^3P^o$  transition in helium using laser excitation of an atomic beam. George *et al.* [2] achieved the most precise result of  $2\ ^3P^o$  fine structure of helium in microwave measurement. Theoretically, Lewis *et al.* [9] predicted the fine structure splittings of the  $J = 1$  to  $J = 0$  interval of the  $1s2p\ ^3P^o$  state by including the major part of the second-order contribution to the fine structure. Drake *et al.* [10–13] substantially improved the precision of the fine structure calculations for this state, with computational uncertainties less than 1 kHz, by including all known terms of order  $\alpha^5$  a.u. ( $\alpha^7 mc^2$ ) arising from the electron–electron interaction and recoil corrections of order  $\alpha^4 \mu/M$  a.u. Using the variational wavefunctions constructed by doubled Hylleraas basis sets, Drake [15] provided essentially exact results for

the entire singly-excited spectrum of helium. However, to our knowledge, few data have been reported for the fine structure of the singly-excited and doubly-excited states of helium atom.

In this paper, the fine structure and the hyperfine structure for some singly-excited and doubly-excited states of helium are calculated using Rayleigh-Ritz variational method with multi-configuration-interaction wave functions. The hyperfine parameters and the hyperfine coupling constants are also explored for this system. The available data should be very useful in better understanding the experimental spectra in the future.

## 2 Theory

The non-relativistic Hamiltonian for helium atom is given in atomic units by

$$H_0 = \sum_{i=1}^2 \left[ -\frac{1}{2} \nabla_i^2 - \frac{2}{r_i} \right] + \frac{1}{r_{12}}. \quad (1)$$

The basic wave function for the two-electron system can be written as

$$\Psi_b(1, 2) = A \sum_i C_i \phi_{n(i), l(i)}(R) Y_{l(i)}^{LM}(\Omega) \chi_{SS_z} \quad (2)$$

where  $A$  is the antisymmetrization operator and the radial basis function is Slater orbital

$$\phi_{n(i), l(i)}(R) = \prod_{j=1}^2 r_j^{n_j} \exp(-\alpha_j r_j). \quad (3)$$

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The angular part is

$$Y_{l(i)}^{LM}(\Omega) = \sum_{m_j} \langle l_1 l_2 m_1 m_2 | LM \rangle \prod_{j=1}^2 Y_{l_j m_j}(\Omega_j) \quad (4)$$

where

$$Y_{lm}(\theta, \varphi) = (-1)^m \left[ \frac{(2l+1)(1-m)!}{4\pi(1+m)!} \right]^{1/2} \times P_l^m(\cos \theta) \exp(im\varphi) \quad (5)$$

and  $P_l^m(\cos \theta)$  is an associated Legendre polynomial.

A different set of  $\alpha_j$  is used for each angular parts  $[l_1, l_2]$ . The linear parameters  $C_i$  and the nonlinear parameters  $\alpha_j$  are determined in the energy optimization processes.

The total energy is further improved by including the relativistic corrections and mass polarization effect. The explicit expressions of these perturbation operators are given in reference [18], they will not be repeated here. Then the total energy becomes  $E_{\text{total}} = E_b + \Delta E_{\text{rel}}$ .

The fine-structure perturbation operators are given by

$$H_{\text{FS}} = H_{\text{SO}} + H_{\text{SOO}} + H_{\text{SS}} \quad (6)$$

where the spin-orbit, spin-other-orbit, and spin-spin operators are

$$H_{\text{SO}} = \frac{Z\alpha^2}{2} \sum_{i=1}^2 \frac{\mathbf{l}_i \cdot \mathbf{s}_i}{r_i^3} \quad (7)$$

$$H_{\text{SOO}} = -\frac{\alpha^2}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^2 \left[ \frac{1}{r_{ij}^3} (\mathbf{r}_i - \mathbf{r}_j) \times \mathbf{P}_i \right] \cdot (\mathbf{s}_i + 2\mathbf{s}_j) \quad (8)$$

$$H_{\text{SS}} = \alpha^2 \sum_{\substack{i,j=1 \\ i \neq j}}^2 \frac{1}{r_{ij}^3} \left[ \mathbf{s}_i \cdot \mathbf{s}_j - \frac{3(\mathbf{s}_i \cdot \mathbf{r}_{ij})(\mathbf{s}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} \right]. \quad (9)$$

The  $\mathbf{l}_i$  and  $\mathbf{s}_i$  are the orbital and the spin angular momentum of the  $i$ th electron. To calculate the fine structure splitting, the  $LSJ$  coupling scheme is used:

$$\Psi_{LSJJZ} = \sum_{S_Z, L_Z} \langle LSL_Z S_Z | JJ_Z \rangle \Psi_b(1, 2). \quad (10)$$

The fine-structure energy levels are calculated by first-order perturbation theory:

$$(\Delta E_{\text{FS}})_J = \langle \Psi_{LSJJZ} | H_{\text{SO}} + H_{\text{SOO}} + H_{\text{SS}} | \Psi_{LSJJZ} \rangle. \quad (11)$$

For a two-electron system, the hyperfine interaction Hamiltonian can be represented as follows [20, 21]:

$$H_{\text{hfs}} = \sum_{k=1} T^{(k)} M^{(k)} \quad (12)$$

where  $T^{(k)}$  and  $M^{(k)}$  are spherical tensor operators of rank  $k$  in the electronic and nuclear space, respectively. The

$k = 1$  term represents the magnetic-dipole interaction between the magnetic field generated by the electrons and nuclear magnetic dipole moments, the  $k = 2$  term the electric quadrupole interaction between the electric field gradient from the electrons and the non-spherical charge distribution of the nucleus. The higher-order contribution terms are much smaller and can often be neglected.

In the non-relativistic framework, the electronic tensor operators can be written as:

$$T^{(1)} = \frac{\alpha^2}{2} \sum_{i=1}^2 \left[ 2g_l r_i^{-3} l_i^{(1)} - \sqrt{10} g_s \left\{ s_i^{(1)} C_i^{(2)} \right\}^{(1)} r_i^{-3} + \frac{8\pi}{3} g_s s_i^{(1)} \delta(\mathbf{r}_i) \right] \quad (13)$$

and

$$T^{(2)} = -\sum_{i=1}^2 r_i^{-3} C_i^{(2)} \quad (14)$$

where  $\alpha$  is the fine structure constants,  $g_l = (1 - m_e/M)$  and  $g_s = 2.0023193$  are the orbital the electron spin  $g$  factor, respectively.  $\delta$  is the three-dimensional delta function.  $M$  is the nuclear mass. The tensor  $C_i^{(2)}$  is connected to the spherical harmonics  $Y_{lm}$  by

$$C_m^{(l)} = \sqrt{\frac{4\pi}{2l+1}} Y_{lm}(\theta, \varphi). \quad (15)$$

The hyperfine interaction couples the electronic angular momenta  $\mathbf{J}$  and the nuclear angular momenta  $\mathbf{I}$  to a total angular momentum  $\mathbf{F} = \mathbf{I} + \mathbf{J}$ . The uncoupling hyperfine constants  $a_C$ ,  $a_{SD}$ ,  $a_l$ , and  $b_q$  are defined as follows [20, 21]:

$$a_C = \left\langle \gamma L S L S \left| \sum_{i=1}^2 8\pi \delta^3(\mathbf{r}_i) s_0(i) \right| \gamma L S L S \right\rangle \quad (\text{Fermi contact}) \quad (16)$$

$$a_{SD} = \left\langle \gamma L S L S \left| \sum_{i=1}^2 2C_0^{(2)}(i) s_0(i) r_i^{-3} \right| \gamma L S L S \right\rangle \quad (\text{spin-dipolar}) \quad (17)$$

$$a_l = \left\langle \gamma L S L S \left| \sum_{i=1}^2 l_0(i) r_i^{-3} \right| \gamma L S L S \right\rangle \quad (\text{orbital}) \quad (18)$$

$$b_q = \left\langle \gamma L S L S \left| \sum_{i=1}^2 2C_0^{(2)}(i) r_i^{-3} \right| \gamma L S L S \right\rangle \quad (\text{electric quadrupole}) \quad (19)$$

and the coupling hyperfine constants  $A_J$  and  $B_J$  are defined as follows:

$$A_J = \frac{\mu_I}{I} \frac{1}{[J(J+1)(2J+1)]^{1/2}} \langle \gamma J || T^{(1)} || \gamma J \rangle \quad (20)$$

$$B_J = 2Q \left[ \frac{2J(2J-1)}{(2J+1)(2J+2)(2J+3)} \right]^{1/2} \langle \gamma J || T^{(2)} || \gamma J \rangle \quad (21)$$

where  $\mu_I$  is the nuclear magnetic moment.  $Q$  is the nuclear electric quadrupole moment.

**Table 1.** Center-of-gravity term energies  $E_{CG}$  ( $\mu\text{a.u.}$ ), fine structure corrections  $\Delta E_J$  (in  $\mu\text{a.u.}$ ) and fine structure splittings  $\nu_{J-J'}$  (in  $\text{cm}^{-1}$ ) for some triplet states in helium.

States	$E_{CG}$	$\Delta E_J$ ( $\mu\text{a.u.}$ )			$\nu_{3-2}$	$\nu_{2-1}$	$\nu_{1-0}$	Source
		$J = L + 1$	$J = L$	$J = L - 1$				
$1s2p$ $^3\text{P}^o$	770 521.69	-0.659	-0.304	4.206		-0.0779	-0.9895	This work
	770 521.69	-0.655	-0.307	4.196		-0.0765	-0.9881	Drake [15]
						-0.0764	-0.9879	Drake [13]
						-0.0764	-0.9879	Zhang and Drake [12]
						-0.0766	-0.9875	Hijikata [26]
						-0.0764	-0.9879	Pchucki [14]
						-0.0764	-0.9879	Exp. [1]
							-0.9879	Exp. [2]
						-0.0764	-0.9879	Exp. [3]
						-0.0764	-0.9879	Exp. [4,6]
$1s3p$ $^3\text{P}^o$	845 618.17	-0.188	-0.076	1.168		-0.0245	-0.2731	This work
	845 610.18	-0.182	-0.181	1.152		-0.0220	-0.2707	Drake [15]
$1s4p$ $^3\text{P}^o$	871 375.61	-0.079	-0.028	0.481		-0.0112	-0.1119	This work
	871 368.26	-0.074	-0.033	0.470		-0.0090	-0.1103	Drake [15]
$1s5p$ $^3\text{P}^o$	883 159.05	-0.042	-0.012	0.247		-0.0066	-0.0570	This work
	883 140.67	-0.037	-0.017	0.236		-0.0045	-0.0554	Drake [15]
$1s6p$ $^3\text{P}^o$	889 543.28	-0.023	-0.005	0.132		-0.0039	-0.0301	This work
	889 484.47	-0.021	-0.010	0.135		-0.0026	-0.0317	Drake [15]
$2p^2$ $^3\text{P}^e$	2 193 289.61	4.480	-4.150	-9.960		1.8932	1.2760	This work
$2p3p$ $^3\text{P}^e$	2 335 973.64	4.400	-4.330	-9.010		1.9165	1.0270	This work
$2p4p$ $^3\text{P}^e$	2 367 920.52	4.428	-4.400	-8.937		1.9372	0.9955	This work
$2p5p$ $^3\text{P}^e$	2 381 534.32	4.437	-4.424	-8.915		1.9444	0.9855	This work
$2p6p$ $^3\text{P}^e$	2 388 627.91	4.442	-4.434	-8.906		1.9477	0.9813	This work
$1s3d$ $^3\text{D}^e$	848 058.65	-0.048	-0.532	0.165	-0.0034	-0.0434		This work
	848 058.65	-0.046	-0.035	0.166	-0.0025	-0.0442		Drake [15]
$1s4d$ $^3\text{D}^e$	872 404.73	-0.020	-0.013	0.070	-0.0015	-0.0183		This work
	872 404.73	-0.020	-0.014	0.070	-0.0012	-0.0185		Drake [15]
$1s5d$ $^3\text{D}^e$	883 673.61	-0.010	-0.007	0.036	-0.0008	-0.0094		This work
	883 673.61	-0.010	-0.007	0.036	-0.0006	-0.0095		Drake [15]
$1s6d$ $^3\text{D}^e$	889 793.21	-0.006	-0.004	0.021	-0.0005	-0.0054		This work
	889 793.21	-0.006	-0.004	0.021	-0.0004	-0.0055		Drake [15]
$2p3d$ $^3\text{D}^o$	2 344 460.22	2.907	-1.452	-4.364	0.9565	0.6392		This work
$2p4d$ $^3\text{D}^o$	2 371 110.79	2.948	-1.471	-4.427	0.9696	0.6488		This work
$2p5d$ $^3\text{D}^o$	2 383 081.68	2.959	-1.477	-4.442	0.9734	0.6505		This work
$2p6d$ $^3\text{D}^o$	2 389 498.99	2.962	-1.480	-4.445	0.9746	0.6508		This work

### 3 Results and discussion

In this work, ten Rydberg series of helium,  $1sns$   $^1,3\text{S}^e$ ,  $1snp$   $^1,3\text{P}^o$ ,  $1snd$   $^1,3\text{D}^e$ ,  $2pnp$   $^1,3\text{P}^e$ , and  $2pnd$   $^1,3\text{D}^o$  states with  $n = 2-6$ , are studied. The fine structure and hyperfine structure for these Rydberg systems are calculated using Rayleigh-Ritz variational method with accurate multi-configuration-interaction wave functions constructed from Slater basis sets. The electron correlation effects between the two electrons are large. Many relevant angular and spin couplings are important for the energy. In order to get the high quality wave function, the number of angular-spin components in the wave functions ranges from 5 to 10, and the number of linear parameters ranges from 85 to 253. The angular series  $[l_1, l_2]$  with more than  $1.0 \times 10^{-5}$  a.u. energy contribution are included in  $\Psi_b$ , and  $l$  is up to 9, as the energy contribution from set with  $l > 9$  is small and negligible.

Table 1 shows the center-of-gravity energies in this work are in good agreement with the theoretical data of Drake [15]. This indicates that the precision of our calculation is sufficiently high in this two-electron system. Comparisons have been made between this work and Drake [15] for the fine structure correction to the center-of-gravity term energy of the  $1snl$   $^3\text{P}^o$ , and  $^3\text{D}^e$  states. Our calculation results of the fine structure corrections are almost the same as that of Drake [15]. This shows that our method is effective in the calculations of the fine structure corrections. The fine structure splittings for this system are also given in Table 1. The  $1s2p$   $^3\text{P}^o$  state is an excellent system to test QED and to measure fine structure constant, many high precision measurements and calculations have been achieved [1-14]. The calculation of the fine structure splittings in  $1s2p$   $^3\text{P}^o$  state has been completed to order  $\alpha^7 mc^2$  by Drake [13]. Compared with these high precision

**Table 2.** Hyperfine parameters of some single-excited and double-excited states for  ${}^3\text{He}$  (in a.u.).

Resonances	$a_c$	$a_{SD}$	$a_l$	$b_q$
$1s2s$ ${}^3S^e$	33.17192			
$1s3s$ ${}^3S^e$	32.28694			
$1s4s$ ${}^3S^e$	32.10615			
$1s5s$ ${}^3S^e$	32.01638			
$1s6s$ ${}^3S^e$	32.02034			
$1s2p$ ${}^1P^o$			0.03825	-0.01214
$1s3p$ ${}^1P^o$			0.01126	-0.00363
$1s4p$ ${}^1P^o$			0.00476	-0.00153
$1s5p$ ${}^1P^o$			0.00248	-0.00079
$1s6p$ ${}^1P^o$			0.00145	-0.00046
$2p3p$ ${}^3P^e$			0.17962	0.07185
$2p4p$ ${}^1P^e$			0.17136	0.06854
$2p5p$ ${}^1P^e$			0.16816	0.06726
$2p6p$ ${}^1P^e$			0.16785	0.06714
$1s2p$ ${}^3P^o$	31.62570	-0.01402	0.06933	-0.02804
$1s3p$ ${}^3P^o$	31.87949	-0.04118	0.02001	-0.00824
$1s4p$ ${}^3P^o$	31.94746	-0.00169	0.00818	-0.00338
$1s5p$ ${}^3P^o$	31.96391	-0.00083	0.00404	-0.00167
$1s6p$ ${}^3P^o$	31.97688	-0.00039	0.00189	-0.00077
$2p^2$ ${}^3P^e$		0.04355	0.21777	0.08711
$2p3p$ ${}^3P^e$		0.03506	0.17531	0.07012
$2p4p$ ${}^3P^e$		0.03399	0.16993	0.06797
$2p5p$ ${}^3P^e$		0.03364	0.16822	0.06729
$2p6p$ ${}^3P^e$		0.03350	0.16751	0.06700
$2p3d$ ${}^1D^o$			0.11430	0.12437
$2p4d$ ${}^1D^o$			0.11251	0.13009
$2p5d$ ${}^1D^o$			0.11177	0.13178
$2p6d$ ${}^1D^o$			0.11148	0.13250
$1s3d$ ${}^1D^e$			0.00499	-0.00121
$1s4d$ ${}^1D^e$			0.00211	-0.00061
$1s5d$ ${}^1D^e$			0.00108	-0.00031
$1s6d$ ${}^1D^e$			0.00062	-0.00018
$2p3d$ ${}^3D^o$		0.06523	0.11462	0.13047
$2p4d$ ${}^3D^o$		0.06606	0.11250	0.13212
$2p5d$ ${}^3D^o$		0.06637	0.11183	0.13275
$2p6d$ ${}^3D^o$		0.06649	0.11151	0.13299
$1s3d$ ${}^3D^e$	31.98370	-0.00072	0.00501	-0.00144
$1s4d$ ${}^3D^e$	31.98610	-0.00030	0.00212	-0.00061
$1s5d$ ${}^3D^e$	31.98722	-0.00016	0.00108	-0.00031
$1s6d$ ${}^3D^e$	31.98759	-0.00009	0.00062	-0.00018

theoretical and experimental data, as Table 1 shows, our results of the fine structure splittings are in reasonable agreement with those of Drake [15]. For example, the fine structure splittings  $\nu_{2-1}$  and  $\nu_{1-0}$  are  $-0.0015 \text{ cm}^{-1}$  and  $-0.0183 \text{ cm}^{-1}$  for  $1s4d$   ${}^3D$  state in this work, respectively. And the corresponding results of Drake [15], using double basis set with Hylleraas basis function, are  $-0.0012 \text{ cm}^{-1}$  and  $-0.0185 \text{ cm}^{-1}$ , respectively. For easy comparison with the results of Drake [15] and experimental data [1–8], we transferred our results to term energies relative to ground state in eV according to the following principle: 1 a.u. = 27.20767 eV, and the energy of the ground state is 79.0056 eV [24]. We transferred the results of Drake [15] from MHz to  $\mu\text{a.u.}$  or  $\text{cm}^{-1}$  according to the following principle: 1 a.u. = 219 444.54  $\text{cm}^{-1}$ . Furthermore, we calculated the fine structure of the doubly-excited

$2lnl$   ${}^3P^e$ , and  ${}^3D^o$  states. To our knowledge, no calculations have been reported for the fine structure of this doubly-excited state system. Our reliable theoretical results should be useful for studying the spectra in the future.

The hyperfine structure is caused by the interaction between the electrons and the electromagnetic multipole moments of the nucleus, sensitive to the correlation effects between electrons and the relativistic corrections. The hyperfine structure of 2  ${}^3P^o$  levels of  ${}^3\text{He}$  has been a subject of experimental and theoretical interest in atomic physics for a long time [25–29]. Recently, the hyperfine splitting of 2  ${}^3S_1$  state in  ${}^3\text{He}$  is explored by Pachucki [30]. A very large second-order correction due to the Fermi interaction have been found in their work. Tables 2 and 3 give the hyperfine parameters and the hyperfine coupling constants

**Table 3.** Hyperfine coupling constants of some single-excited and double-excited states for  ${}^3\text{He}$  (in GHz).

Resonances	$A_J$		
	$J = 3$	$J = 2$	$J = 1$
$1s2s\ {}^3\text{S}^e$			-4.49443
$1s3s\ {}^3\text{S}^e$			-4.37453
$1s4s\ {}^3\text{S}^e$			-4.35003
$1s5s\ {}^3\text{S}^e$			-4.34193
$1s6s\ {}^3\text{S}^e$			-4.33840
$1s2p\ {}^1\text{P}^o$			-1.55276(-2)
$1s3p\ {}^1\text{P}^o$			-4.57232(-3)
$1s4p\ {}^1\text{P}^o$			-1.93333(-3)
$1s5p\ {}^1\text{P}^o$			-1.00639(-3)
$1s6p\ {}^1\text{P}^o$			-5.86835(-4)
$2p3p\ {}^1\text{P}^e$			-7.29127(-2)
$2p4p\ {}^1\text{P}^e$			-6.95572(-2)
$2p5p\ {}^1\text{P}^e$			-6.82588(-2)
$2p6p\ {}^1\text{P}^e$			-6.81350(-2)
$1s2p\ {}^3\text{P}^o$		-2.15369	-2.17079
$1s3p\ {}^3\text{P}^o$		-2.16288	-2.16791
$1s4p\ {}^3\text{P}^o$		-2.16558	-2.16764
$1s5p\ {}^3\text{P}^o$		-2.16603	-2.16705
$1s6p\ {}^3\text{P}^o$		-2.16656	-2.16704
$2p^2\ {}^3\text{P}^o$		-5.30509(-2)	5.93094(-5)
$2p3p\ {}^3\text{P}^e$		-4.27064(-2)	4.77446(-5)
$2p4p\ {}^3\text{P}^e$		-4.13974(-2)	4.62811(-5)
$2p5p\ {}^3\text{P}^e$		-4.09792(-2)	4.58136(-5)
$2p6p\ {}^3\text{P}^e$		-4.08069(-2)	4.56210(-5)
$2p3d\ {}^1\text{D}^o$		-2.31993(-2)	
$2p4d\ {}^1\text{D}^o$		-2.28348(-2)	
$2p5d\ {}^1\text{D}^o$		-2.26855(-2)	
$2p6d\ {}^1\text{D}^o$		-2.26271(-2)	
$1s3d\ {}^1\text{D}^e$		-1.01304(-3)	
$1s4d\ {}^1\text{D}^e$		-4.27647(-4)	
$1s5d\ {}^1\text{D}^e$		-2.18655(-4)	
$1s6d\ {}^1\text{D}^e$		-1.26719(-4)	
$2p3d\ {}^3\text{D}^o$	-2.43477(-2)	-3.91885(-3)	1.15071(-2)
$2p4d\ {}^3\text{D}^o$	-2.41731(-2)	-3.36418(-3)	1.27410(-2)
$2p5d\ {}^3\text{D}^o$	-2.41240(-2)	-3.17616(-3)	1.31681(-2)
$2p6d\ {}^3\text{D}^o$	-2.40974(-2)	-3.09421(-3)	1.33497(-2)
$1s3d\ {}^3\text{D}^e$	-1.44506	-7.23259(-1)	2.16468
$1s4d\ {}^3\text{D}^e$	-1.44483	-7.22724(-1)	2.16602
$1s5d\ {}^3\text{D}^e$	-1.44476	-7.22540(-1)	2.16652
$1s6d\ {}^3\text{D}^e$	-1.44473	-7.22455(-1)	2.16673

$A_J$  for  $1sns\ {}^1,{}^3\text{S}^e$ ,  $1snp\ {}^1,{}^3\text{P}^o$ ,  $1snd\ {}^1,{}^3\text{D}^e$ ,  $2pnp\ {}^1,{}^3\text{P}^e$ , and  $2pnd\ {}^1,{}^3\text{D}^o$  states in  ${}^3\text{He}$ . In this calculation, high precise wave functions are used. We studied the hyperfine structure parameters: Fermi contact  $a_c$ , the spin-dipolar  $a_{\text{SD}}$ , the orbital  $a_l$ , and the electric quadrupole  $b_q$ . In this work,  $Q = 0.0b$ ,  $\mu_I = -2.127625$  nm, and  $I = 1/2$  are taken from reference [19]. To our knowledge, few data of the hyperfine parameters  $a_c$ ,  $a_{\text{SD}}$ ,  $a_l$ ,  $b_q$ , and hyperfine coupling constants  $A_J$  have been reported for this system of  ${}^3\text{He}$  in the literature.

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